TABLE A6.1 *Fourier-Transform Theorems*

Property	*Mathematical Description*
1. Linearity	$ag_1(t) + bg_2(t) \rightleftharpoons aG_1(f) + bG_2(f)$ where a and b are constants
2. Dilation (time scaling)	$g(at) \rightleftharpoons \frac{1}{\|a\|}G\left(\frac{f}{a}\right)$ where a is a constant
3. Duality	If $g(t) \rightleftharpoons G(f)$, then $G(t) \rightleftharpoons g(-f)$
4. Time shifting	$g(t - t_0) \rightleftharpoons G(f)\exp(-j2\pi f t_0)$
5. Frequency shifting	$\exp(j2\pi f_c t)g(t) \rightleftharpoons G(f - f_c)$
6. Area under $g(t)$	$\int_{-\infty}^{\infty} g(t)\,dt = G(0)$
7. Area under $G(f)$	$g(0) = \int_{-\infty}^{\infty} G(f)\,df$
8. Differentiation in the time domain	$\frac{d}{dt}g(t) \rightleftharpoons j2\pi f G(f)$
9. Integration in the time domain	$\int_{-\infty}^{t} g(\tau)\,d\tau \rightleftharpoons \frac{1}{j2\pi f}G(f) + \frac{G(0)}{2}\delta(f)$
10. Conjugate functions	If $g(t) \rightleftharpoons G(f)$, then $g^*(t) \rightleftharpoons G^*(-f)$,
11. Multiplication in the time domain	$g_1(t)g_2(t) \rightleftharpoons \int_{-\infty}^{\infty} G_1(\lambda)G_2(f-\lambda)\,d\lambda$
12. Convolution in the time domain	$\int_{-\infty}^{\infty} g_1(\tau)g_2(t-\tau)\,d\tau \rightleftharpoons G_1(f)G_2(f)$
13. Correlation theorem	$\int_{-\infty}^{\infty} g_1(t)g_2^*(t-\tau)\,dt \rightleftharpoons G_1(f)G_2^*(f)$
14. Rayleigh's energy theorem	$\int_{-\infty}^{\infty} \|g(t)\|^2\,dt = \int_{-\infty}^{\infty} \|G(f)\|^2\,df$

TABLE A6.2 *Fourier-Transform Pairs*

Time Function	*Fourier Transform*
$\mathrm{rect}\left(\frac{t}{T}\right)$	$T\,\mathrm{sinc}\,(fT)$
$\mathrm{sinc}\,(2Wt)$	$\frac{1}{2W}\mathrm{rect}\left(\frac{f}{2W}\right)$
$\exp(-at)u(t), \quad a > 0$	$\frac{1}{a + j2\pi f}$
$\exp(-a\|t\|), \quad a > 0$	$\frac{2a}{a^2 + (2\pi f)^2}$
$\exp(-\pi t^2)$	$\exp(-\pi f^2)$
$\begin{cases} 1 - \frac{\|t\|}{T}, & \|t\| < T \\ 0, & \|t\| \geq T \end{cases}$	$T\,\mathrm{sinc}^2(fT)$
$\delta(t)$	1
1	$\delta(f)$
$\delta(t - t_0)$	$\exp(-j2\pi f t_0)$
$\exp(j2\pi f_c t)$	$\delta(f - f_c)$
$\cos(2\pi f_c t)$	$\frac{1}{2}[\delta(f - f_c) + \delta(f + f_c)]$
$\sin(2\pi f_c t)$	$\frac{1}{2j}[\delta(f - f_c) - \delta(f + f_c)]$
$\mathrm{sgn}(t)$	$\frac{1}{j\pi f}$
$\frac{1}{\pi t}$	$-j\,\mathrm{sgn}(f)$
$u(t)$	$\frac{1}{2}\delta(f) + \frac{1}{j2\pi f}$
$\sum_{i=-\infty}^{\infty} \delta(t - iT_0)$	$\frac{1}{T_0}\sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T_0}\right)$

Notes: $u(t)$ = unit step function
$\delta(t)$ = Dirac delta function
$\mathrm{rect}(t)$ = rectangular function
$\mathrm{sgn}(t)$ = signum function
$\mathrm{sinc}(t)$ = sinc function

TABLE A6.4 *Trigonometric Identities*

$\exp(\pm j\theta) = \cos\theta \pm j\sin\theta$
$\cos\theta = \frac{1}{2}[\exp(j\theta) + \exp(-j\theta)]$
$\sin\theta = \frac{1}{2j}[\exp(j\theta) - \exp(-j\theta)]$
$\sin^2\theta + \cos^2\theta = 1$
$\cos^2\theta - \sin^2\theta = \cos(2\theta)$
$\cos^2\theta = \frac{1}{2}[1 + \cos(2\theta)]$
$\sin^2\theta = \frac{1}{2}[1 - \cos(2\theta)]$
$2\sin\theta\cos\theta = \sin(2\theta)$
$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$
$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$
$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$
$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$
$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)]$
$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha - \beta) + \sin(\alpha + \beta)]$

Energy signals

$$R_x(\tau) = \int_{-\infty}^{+\infty} x(t)x^*(t-\tau)dt = x(t) * x^*(-t)\big|_{t=\tau}$$

$$\Psi_x(f) = |X(f)|^2 = \mathcal{F}\{R_x(\tau)\}$$

$$E_x = \int_{-\infty}^{+\infty} \Psi_x(f)df$$

Power signals

$$R_x(\tau) = \lim_{T_0\to+\infty} \frac{1}{2T_0}\int_{-T_0}^{T_0} x(t)x^*(t-\tau)dt$$

$$S_x(f) = \mathcal{F}\{R_x(\tau)\}$$

$$P_x = \int_{-\infty}^{+\infty} S_x(f)df$$

Periodic signals

$$R_x(\tau) = \frac{1}{T}\int_0^T x(t)x^*(t-\tau)dt = \sum_{n=-\infty}^{+\infty} |x_n|^2 \exp(j2\pi nt/T)$$

$$S_x(f) = \sum_{n=-\infty}^{+\infty} |x_n|^2\, \delta\left(f - \frac{n}{T}\right)$$

White noise

$$S_w(f) = \frac{N_0}{2}$$

PAM signals

$$x(t) = \sum_k a_k r(t - kT)$$

$$S_x(f) = \frac{1}{T}\left[\overline{a_k^2} - (\overline{a}_k)^2\right]|R(f)|^2 + \frac{(\overline{a}_k)^2}{T^2}\sum_n \left|R\left(\frac{n}{T}\right)\right|^2 \delta\left(f - \frac{n}{T}\right)$$