EXAMPLE 1 GRAPHICAL INTERPRETATION OF CONVOLUTION

We may develop further insight into convolution by presenting a graphical interpretation of the convolution integral, which is defined in mathematical terms in Eq. 3.1 or 3.2. We will do so in this example by considering Eq. 3.1 first and then 3.2. The example is simple and yet illustrative of the various steps involved in evaluating the convolution integral. Specifically, we consider a linear time-invariant system with an impulse response that is a decaying exponential function and that is driven by a unit step function.

Parts a and b of Fig. 3.2 depict the impulse response $h(\tau)$ and excitation

(a)
(b)
(c)

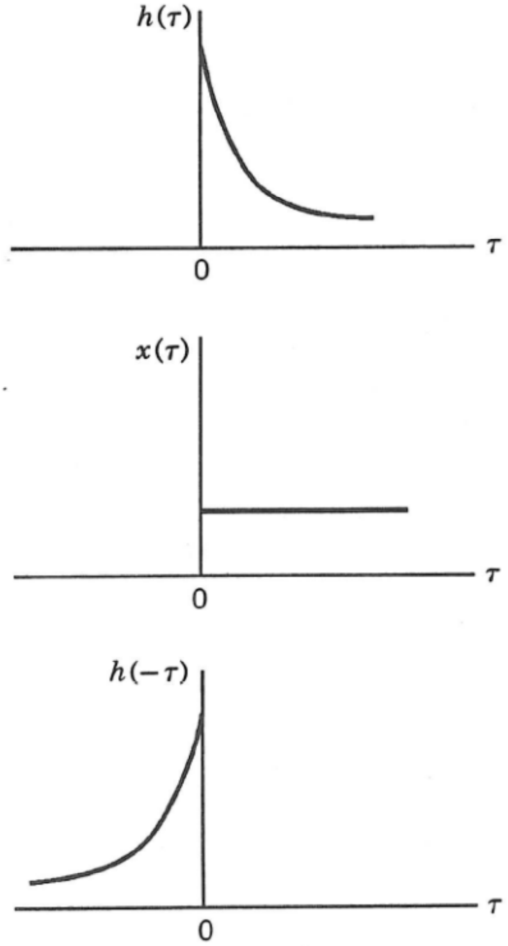

Figure 3.2
The steps involved in computing one form of the convolution integral. (a) Impulse response. (b) Excitation. (c) Image of the impulse response. (d) Time-shifted image of the impulse response. (e) Evaluation of the response.

(d)

(e)

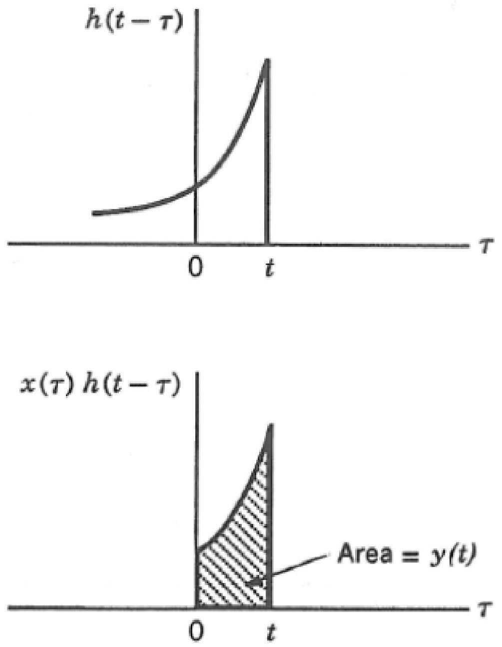

Figure 3.2 (*continued*)

$x(\tau)$, respectively. For reasons that will become apparent presently, the time variable in both cases is shown as τ. In accordance with Eq. 3.1, the integral consists of the product $x(\tau)h(t - \tau)$. We already have $x(\tau)$. To obtain $h(t - \tau)$, we proceed in two steps. First, we formulate $h(-\tau)$, which is the mirror image of $h(\tau)$ with respect to the vertical axis, as shown in Fig. 3.2*c*. Then, we shift $h(-\tau)$ to the right by an amount equal to the specified time t to obtain $h(t - \tau)$; this second step is shown in Fig. 3.2*d*. Next, we multiply $x(\tau)$ by $h(t - \tau)$, as in Fig. 3.2*e*, and thereby obtain the desired integrand $x(\tau)h(t - \tau)$ for the specified value of time t. Finally, we calculate the total area under $x(\tau)h(t - \tau)$, which is shown shaded in Fig. 3.2*e*. This area equals the value of the system response $y(t)$ at time t.

For the graphical interpretation of Eq. 3.2 we may proceed in a similar way, as illustrated in Fig. 3.3. In this second case, the integrand equals $h(\tau)x(t - \tau)$. The first multiplying factor $h(\tau)$ is already available, as in Fig. 3.3*a*. The second multiplying factor $x(t - \tau)$ is obtained by forming the image $x(-\tau)$ of the specified excitation $x(\tau)$, and then shifting the image $x(-\tau)$ to the right by an amount equal to the specified time t. The functions $x(\tau)$, $x(-\tau)$, and $x(t - \tau)$ are depicted in Figs. 3.3*b*, *c*, and *d*, respectively. The resulting product $h(\tau)x(t - \tau)$ is shown in Fig. 3.3*e*. Comparing Figs. 3.2*e* and 3.3*e*, we see that the products $x(\tau)h(t - \tau)$ and $h(\tau)x(t - \tau)$ are reversed with respect to each other. Naturally, they both have the same total area under their individual curves, which confirms the commutative property of convolution.

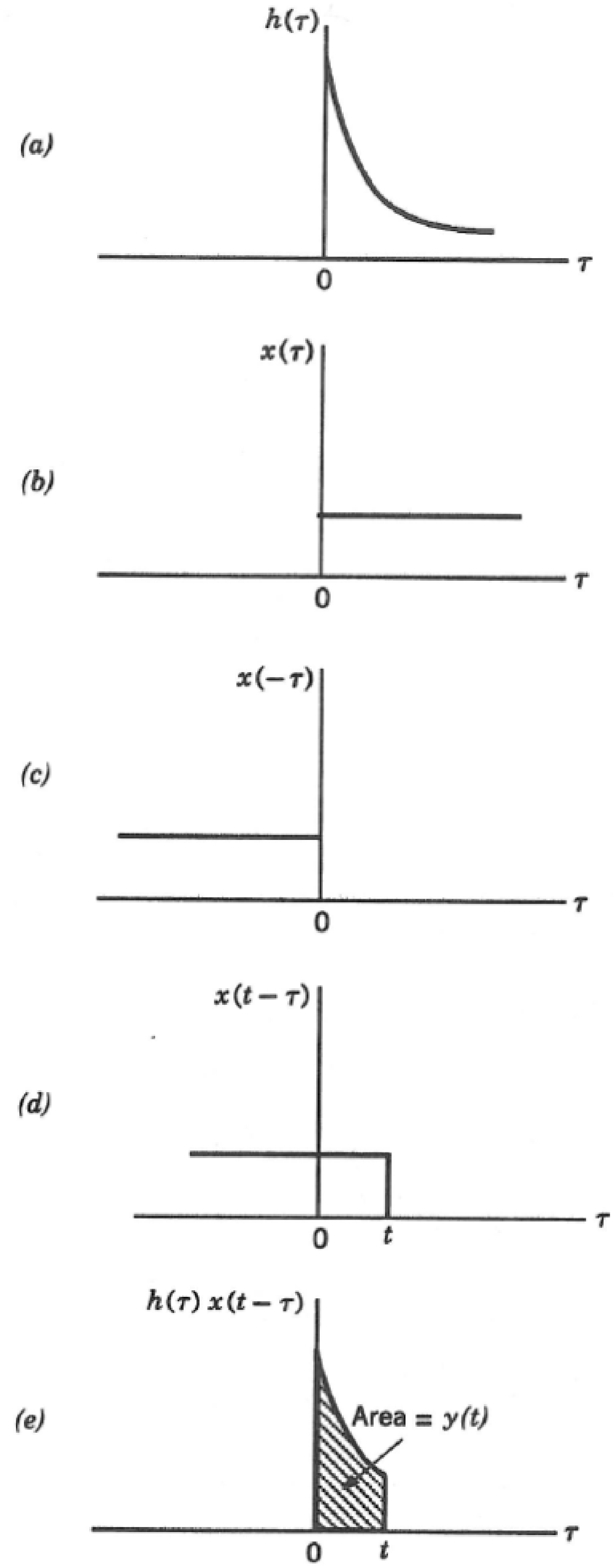

Figure 3.3
The steps involved in computing the second form of the convolution integral. (a) Impulse response. (b) Excitation. (c) Image of the excitation. (d) Time-shifted image of the excitation. (e) Evaluation of the response y(t).