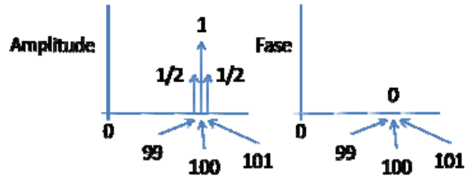
1º Teste 2017/2018



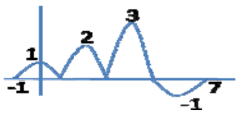
1. a)

$$x(t)=\cos(200\pi t)+\frac{1}{2}\cos(202\pi t)+\frac{1}{2}\cos(198\pi t)$$

   b)

$$y(t)=\cos(200\pi t)$$

2. a)



   b) Não causal, pois $h(t)$ começa em –1. Estável, pois $\int_{-\infty}^{+\infty}|h(t)|\,dt<2T<\infty$

3. a) $R_b=2Bv\Rightarrow v=10\Rightarrow L=2^{v}=1024$

   b) $v=5\Rightarrow L=32=\sqrt{1024}$

4. a) $T/T_0=0.2=1/5,\quad P_x=A^2T/T_0=1\quad\Rightarrow\quad A=\sqrt{5}$

   b) $x_{DC}=x_0=AT/T_0=1/\sqrt{5}$

   c) $x_n=AT/T_0\mathrm{sinc}(nT/T_0)=\mathrm{sinc}(n/5)/\sqrt{5}$

2º Teste 2017/2018

1.

$$Y(f)=j2\pi fX(f)$$

$$\int_{-\infty}^{+\infty}y(t)dt=Y(0)=j2\pi fX(f)\big|_{f=0}=0,\ \text{pois}\ |X(f)|<M$$

2.

$$y(t)=x(t)*A=\int_{-\infty}^{+\infty}x(\tau)\mathrm{A}dt=AX(0)$$

(alternativa) $Y(f)=X(f)A\delta(f)=X(0)A\delta(f)\rightarrow y(t)=AX(0)$

3.

$$H(f)=kf\Rightarrow Y(f)=X(f)H(f)=kfX(f)$$

$$x_{DC}\neq 0\Rightarrow X(f)=x_{DC}\delta(f)+\underbrace{X_1(f)}_{\text{Sem Diracs na origem}}$$

$$Y(f)=kfX(f)=kfx_{DC}\delta(f)+kfX_1(f)=kfX_1(f)\Rightarrow\text{Sem Diracs na origem}\Rightarrow y_{DC}=0$$

4. $X(f)=\mathrm{rect}((f-1/T)T)/2+\mathrm{rect}((f+1/T)T)/2=\mathrm{rect}(fT/2))/2\Rightarrow B=1/T\Rightarrow F_a=2B=2/T$

5. a) $F_a=2B=1MHz\Rightarrow T_a=1/F_a=1\mu s$

   b)

   $x(t_n)=0\Rightarrow$ Não se consegue recuperar o sinal original

   Embora F_a=2B, não se contradiz o teorama da amostragem pois o espectro acaba num Dirac

   c) $\Delta=2/2^{v}\Rightarrow P_N=4^{-v}/3\Rightarrow SNR=P_x/P_N=P_x3\cdot 4^{v}$

   d) Mais um bit de quantização dá um ganho dum factor de 4 em SNR, ou seja +6dB

   e) $R_b=F_av=2Bv=8Mbps\quad\Rightarrow\quad B_{\text{Canal}}=R_b/\varepsilon=4MHz$ pois $\varepsilon_{\text{(polar binário)}}=2/(1+\rho)\leq 2$

3º Teste 2017/2018

1.

$$x'(t) = -At\sigma^2 e^{-t^2/2\sigma^2}$$

$$x''(t) = At^2\sigma^4 e^{-t^2/2\sigma^2} - A\sigma^2 e^{-t^2/2\sigma^2} = 0 \Rightarrow t = \pm 1/\sigma \Rightarrow \max(|x'(t)|) = |x'(1/\sigma)|$$

$$\Delta / T_a = \Delta F_a > \max(|x'(t)|) \Rightarrow F_a > \max(|x'(t)|)/\Delta$$

2.

a) $a_n + jb_n$ corresponde a quadrado 4x4 $\Rightarrow x(t)$ 16-QAM

b)

$$S_{x_I}(f) = S_{x_Q}(f) = \frac{\overline{a_n^2}}{T}|R(f)|^2 = 5A^2 T\mathrm{rect}(fT)$$

$$\Rightarrow S_x(f) = \frac{1}{4}S_{x_I}(f-f_c) + \frac{1}{4}S_{x_I}(f+f_c) + \frac{1}{4}S_{x_Q}(f-f_c) + \frac{1}{4}S_{x_Q}(f+f_c) =$$

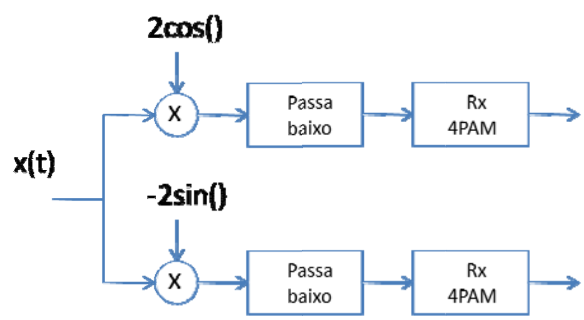
$$= \frac{1}{2}S_{x_I}(f-f_c) + \frac{1}{2}S_{x_I}(f+f_c)$$

c) $\varepsilon_{16\mathrm{QAM}} = \dfrac{4}{1+\rho} \Rightarrow R_{b,\max} = 4B = 4Mbps$

d)

e) $a_n + jb_n = A\sqrt{2}e^{j\varphi_\kappa}$ com $\varphi_\kappa = \pm\pi/4, \pm 3\pi/4 \Rightarrow x(t)$ QPSK

3. a) $B_{FM} \approx \begin{cases} 2Ak_f, & \text{polar} \\ Ak_f, & \text{unipolar} \end{cases}$



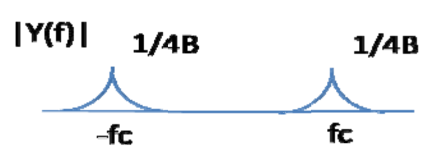
b) Sinal FM contínuo. Frequência instantânea segue o andamento da mensagem, logo é descontínua.

Exame 2017/2018

1. a)

$$X(f) = \frac{1}{a + j2\pi f}$$

$$Y(f) = \frac{1}{2}X(f-f_c) + \frac{1}{2}X(f+f_c)$$



$$|X(f)| = \frac{1}{\sqrt{a^2 + (2\pi f)^2}}$$

$$|Y(f)| \approx \frac{1}{2}\frac{1}{\sqrt{a^2 + (2\pi(f-f_c))^2}} + \frac{1}{2}\frac{1}{\sqrt{a^2 + (2\pi(f+f_c))^2}}$$

b)

$$E_x = \int_{-\infty}^{+\infty} x^2(t)dt = \int_0^{+\infty} e^{-2at}dt = \frac{1}{2a}$$
$$E_y = E_x / 2$$

c)
$$|X(B_{x,3dB})|^2 = \frac{1}{a^2 + (2\pi B_{x,3dB})^2} = \frac{|X(0)|^2}{2} = \frac{1}{2a^2} \Rightarrow B_{x,3dB} = \frac{a}{2\pi}$$
$$B_{y,3dB} = 2B_{x,3dB} = \frac{a}{\pi}$$

d)
$$B_{y,99\%} = 2B_{x,99\%}$$

2.

a) $F_a = 2B = 50kbps,\ L = 64 = 2^{\nu} \Rightarrow \nu = 6 \Rightarrow R_b = F_a\nu = 300kHz$

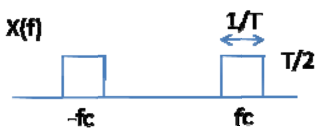
b) $\varepsilon = \frac{2\log_2(M)}{1+\rho} = \frac{R_b}{B} = 12 \Rightarrow M = 64 \quad (64-PAM)$

3.
$$X(f) = \sum_n x_n \delta(f - n/T), \qquad H(f) = \text{sinc}^2(fT)$$
$$Y(f) = X(f)H(f) = \sum_n x_n H(n/T)\delta(f - n/T) = x_0\delta(f) \rightarrow y(t) = x_0 = x_{DC}$$

4.

a) B=1/T

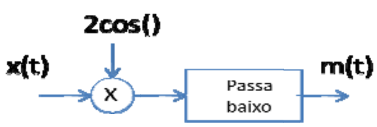
b)



c)
$$X(f) = \frac{1}{2}M(f - f_c) + \frac{1}{2}M(f + f_c) \Rightarrow M(f) = T\text{rect}(fT) \Rightarrow m(t) = \text{sinc}(t/T)$$

d) Não, pois $m(t)$ pode ser >0 ou <0

e) Receptor coerente:

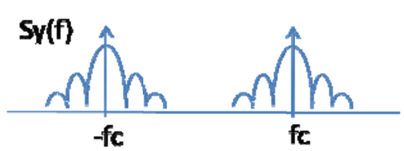


5.

a)
$\varepsilon_{\max,4-APSK} = 2$ (constelação com 4 pointos (2 bits) + DSB)

b) Sim, pois a mensagem só afecta a amplitude.

c)
$$x(t) = \underbrace{\sum_n a_n r(t - nT)}_{x_I(t)} \cos(2\pi f_c t)$$



$$\bar{a}_n = 3A/2, \quad \overline{a_n^2} = 7A^2/2, \quad R(f) = T\text{sinc}(fT), \quad R(n/T) = T\text{sinc}(n)$$
$$S_{x_I}(f) = \frac{\overline{a_n^2} - (\bar{a}_n)^2}{T}|R(f)|^2 + (\bar{a}_n)^2\delta(f) = \frac{5A^2}{4}T\text{sinc}^2(fT) + \frac{9A^2}{4}\delta(f)$$
$$S_x(f) = \frac{S_{x_I}(f - f_c) + S_{x_I}(f + f_c)}{4}$$